

10-4

Practice

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. $\log 101$

2. $\log 2.2$

3. $\log 0.05$

Use the formula $\text{pH} = -\log[H^+]$ to find the pH of each substance given its concentration of hydrogen ions.

4. milk: $[H^+] = 2.51 \times 10^{-7}$ mole per liter

5. acid rain: $[H^+] = 2.51 \times 10^{-6}$ mole per liter

6. black coffee: $[H^+] = 1.0 \times 10^{-5}$ mole per liter

7. milk of magnesia: $[H^+] = 3.16 \times 10^{-11}$ mole per liter

Solve each equation or inequality. Round to four decimal places.

8. $2^x < 25$

9. $5^a = 120$

10. $6^z = 45.6$

11. $9^m \geq 100$

12. $3.5^x = 47.9$

13. $8.2^y = 64.5$

14. $2^{b+1} \leq 7.31$

15. $4^{2x} = 27$

16. $2^a - 4 = 82.1$

17. $9^x - 2 > 38$

18. $5^{w+3} = 17$

19. $30^{x^2} = 50$

20. $5^{x^2-3} = 72$

21. $4^{2x} = 9^x + 1$

22. $2^{n+1} = 5^{2n-1}$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_5 12$

24. $\log_8 32$

25. $\log_{11} 9$

26. $\log_2 18$

27. $\log_9 6$

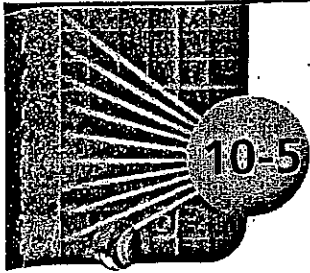
28. $\log_7 \sqrt{8}$

29. **HORTICULTURE** Siberian irises flourish when the concentration of hydrogen ions $[H^+]$ in the soil is not less than 1.58×10^{-8} mole per liter. What is the pH of the soil in which these irises will flourish?

30. **ACIDITY** The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk?

31. **BIOLOGY** There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria N present after t hours is $N = 1000(2)^t$. How long will it take the culture to increase to 50,000 bacteria?

32. **SOUND** An equation for loudness L in decibels is given by $L = 10 \log R$, where R is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise?



NAME _____

DATE _____

PERIOD _____

Skills Practice

Base e and Natural Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. e^3

2. e^{-2}

3. $\ln 2$

4. $\ln 0.09$

Write an equivalent exponential or logarithmic equation.

5. $e^x = 3$

6. $e^4 = 8x$

7. $\ln 15 = x$

8. $\ln x \approx 0.6931$

Evaluate each expression.

9. $e^{\ln 8}$

10. $e^{\ln 2x}$

11. $\ln e^{-2.5}$

12. $\ln e^y$

Solve each equation or inequality.

13. $e^x \geq 5$

14. $e^x < 3.2$

15. $2e^x - 1 = 11$

16. $5e^x + 3 = 18$

17. $e^{3x} = 30$

18. $e^{-4x} > 10$

19. $e^{5x} + 4 > 34$

20. $1 - 2e^{2x} = -19$

21. $\ln 3x = 2$

22. $\ln 8x = 3$

23. $\ln (x - 2) = 2$

24. $\ln (x + 3) = 1$

25. $\ln (x + 3) = 4$

26. $\ln x + \ln 2x = 2$



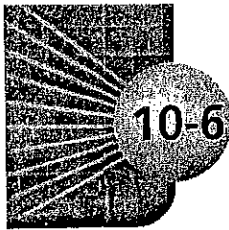
10-6

Skills Practice

Exponential Growth and Decay

Solve each problem.

1. **FISHING** In an over-fished area, the catch of a certain fish is decreasing at an average rate of 8% per year. If this decline persists, how long will it take for the catch to reach half of the amount before the decline?
2. **INVESTING** Alex invests \$2000 in an account that has a 6% annual rate of growth. To the nearest year, when will the investment be worth \$3600?
3. **POPULATION** A current census shows that the population of a city is 3.5 million. Using the formula $P = ae^{rt}$, find the expected population of the city in 30 years if the growth rate r of the population is 1.5% per year, a represents the current population in millions, and t represents the time in years.
4. **POPULATION** The population P in thousands of a city can be modeled by the equation $P = 80e^{0.015t}$, where t is the time in years. In how many years will the population of the city be 120,000?
5. **BACTERIA** How many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%?
6. **NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of k for this element?
7. **DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for \$12,500. The GPS depreciated by a fixed rate of 6% and is now worth \$8600. How long ago did Abu buy the GPS system?
8. **BIOLOGY** In a laboratory, an organism grows from 100 to 250 in 8 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$?

**10-6 Study Guide and Intervention** (continued)**Exponential Growth and Decay**

Exponential Growth Population increase and growth of bacteria colonies are examples of **exponential growth**. When a quantity increases by a fixed percent each time period, the amount of that quantity after t time periods is given by $y = a(1 + r)^t$, where a is the initial amount and r is the percent increase (or rate of growth) expressed as a decimal.

Another exponential growth model often used by scientists is $y = ae^{kt}$, where k is a constant.

Example

A computer engineer is hired for a salary of \$28,000. If she gets a 5% raise each year, after how many years will she be making \$50,000 or more?

Use the exponential growth model with $a = 28,000$, $y = 50,000$, and $r = 0.05$ and solve for t .

$$y = a(1 + r)^t$$

Exponential growth formula

$$50,000 = 28,000(1 + 0.05)^t$$

$$y = 50,000, a = 28,000, r = 0.05$$

$$\frac{50}{28} = (1.05)^t$$

Divide each side by 28,000.

$$\log\left(\frac{50}{28}\right) = \log(1.05)^t$$

Property of Equality of Logarithms

$$\log\left(\frac{50}{28}\right) = t \log 1.05$$

Power Property

$$t = \frac{\log\left(\frac{50}{28}\right)}{\log 1.05}$$

Divide each side by $\log 1.05$.

$$t \approx 11.9 \text{ years}$$

Use a calculator.

If raises are given annually, she will be making over \$50,000 in 12 years.

Exercises

- 1. BACTERIA GROWTH** A certain strain of bacteria grows from 40 to 326 in 120 minutes. Find k for the growth formula $y = ae^{kt}$, where t is in minutes.
- 2. INVESTMENT** Carl plans to invest \$500 at 8.25% interest, compounded continuously. How long will it take for his money to triple?
- 3. SCHOOL POPULATION** There are currently 850 students at the high school, which represents full capacity. The town plans an addition to house 400 more students. If the school population grows at 7.8% per year, in how many years will the new addition be full?
- 4. EXERCISE** Hugo begins a walking program by walking $\frac{1}{2}$ mile per day for one week. Each week thereafter he increases his mileage by 10%. After how many weeks is he walking more than 5 miles per day?
- 5. VOCABULARY GROWTH** When Emily was 18 months old, she had a 10-word vocabulary. By the time she was 5 years old (60 months), her vocabulary was 2500 words. If her vocabulary increased at a constant percent per month, what was that increase?